American University of Beirut
MATH 201
Calculus and Analytic Geometry III
Fall 2009-2010

Final Exam

Exercise 1 a. *(3 points)* Find the directional derivative of $f(x, y) = x^2e^{-2y}$ at $P(1, 0)$ in the direction of the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$

b. *(3 points)* The equation $1 - x - y^2 - \sin(xy)$ defines y as a differentiable function of x. Find dy/dx at the point $P(0, 1)$.

c. *(5 points)* Find the points on the surface $xy + yz + zx - x - z^2 = 0$, where the tangent plane is parallel to the xy-plane.

Exercise 2 *(12 points)* Find the absolute minimum and maximum values of $f(x, y) = x^2 + xy + y^2 - 3x + 3y$ on the triangular region R cut from the first quadrant by the line $x + y = 4$.

Exercise 3 *(6 points)* Use Lagrange Multipliers to find the maximum and the minimum values of $f(x, y) = xy$ subject to the constraint $x^2 + y^2 = 10$.

Exercise 4 *(10 points)* Reverse the order of integration, then evaluate the integral

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx$$

Exercise 5 *(10 points)* Let V be the volume of the region D that is bounded below by the xy-plane, above by the paraboloid $z = 9 - x^2 - y^2$, and lying outside the cylinder $x^2 + y^2 = 1$.

a. express V as an iterated triple integral in cylindrical coordinates, then *evaluate* the resulting integral.

b. express, but *do not evaluate* , V as an iterated triple integral in cylindrical coordinates in the order $drdzd\theta$.

Exercise 6 *(10 points)* Let V be the volume of the region that is bounded form below by the sphere $x^2 + y^2 + (z - 1)^2 = 1$ and from above by the cone $z = \sqrt{x^2 + y^2}$. Express V as an iterated triple integral in spherical coordinates, then evaluate the resulting integral *(sketch the region of integration)*.

Exercise 7 *(12 points)* Write six different iterated triple integrals for the volume of the tetrahedron cut from the first octant by the plane $6x + 3y + 2z = 6$.
(do not evaluate any of the integrals)

⟶

Exercise 8 a. *(6 points)* Find the line integral of $f(x,y) = (x+y^2)/\sqrt{1+x^2}$ along the curve $C : y = x^2/2$ from $(1, 1/2)$ to $(0,0)$.

b. *(8 points)* Show that the differential form $2\cos y dx + (\frac{1}{y} - 2x\sin y)dy + (1/z)dz$ is exact, then evaluate

$$\int_{(0,2,1)}^{(1,\pi/2,2)} 2\cos y dx + \left(\frac{1}{y} - 2x\sin y\right) dy + (1/z)dz$$

c. Find the *counterclockwise circulation* of the field $F = 2xy^3\mathbf{i} + 4x^2y^2\mathbf{j}$ across the curve C in the first quadrant, bounded by the lines $y = 0$, $x = 1$ and the curve $y = x^3$.

i. *(8 points)* by direct calculation

ii. *(8 points)* by Green's theorem

good luck